Math 290 A Friday, March 13

V - vecter space $\quad S = \{ \underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_m} \}$

$\langle S \rangle$ all linear combinations of vecters in S

<u>Ex</u> P_3 - all polys w/ degree 3 or less (P_n)

$S = \{ 1 - 2x + x^2 + 3x^3, 2 + 3x^2 + x^3 \}$, $-3 - 2x + 4x^2 + x^3 \in \langle S \rangle$

$$-3 - 2x + 4x^2 + x^3 = a(1 - 2x + x^2 + 3x^3) + b(2 + 3x^2 + x^3)$$

$$= a - 2ax + ax^2 + 3ax^3 + 2b + 3bx^2 + bx^3$$

$$= (a + 2b) + (-2a)x + (a + 3b)x^2 + (3a + b)x^3$$

$a + 2b = -3$
$-2a = -2$
$a + 3b = 4$
$3a + b = 1$

augmented matrix $\xrightarrow{\cancel{RREF}}$ $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$

RCLS $\Rightarrow$ no solution

no a, b

so $\notin \langle S \rangle$

Ex M_{22} $T = \left\{ \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \right\}$ T linearly independent?

$$a \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} + b \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} + c \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} = \underset{\sim}{0}$$

$$\begin{bmatrix} 2a+b+2c & a+c \\ -a+3b+c & 2a+b-c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 2a+b+2c &= 0 \\ a+c &= 0 \\ -a+3b+c &= 0 \\ 2a+b-c &= 0 \end{aligned}$$

augmented matrix $\xrightarrow{RREF}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

So only solution is $a=b=c=0$

So T is linearly independent.

Theorem $N(A)$ is a subspace. ← of $\mathbb{C}^n$

$\uparrow_{m\times n}$

Proof 1) $\underset{\sim}{0} \in N(A)$? $\quad A\underset{\sim}{0} = \underset{\sim}{0} \Rightarrow \underset{\sim}{0} \in N(A) \Rightarrow N(A) \neq \emptyset$

Closure

2) ~~If~~ Suppose $\underset{\sim}{x}, \underset{\sim}{y} \in N(A)$. Know $A\underset{\sim}{x} = \underset{\sim}{0}$, $A\underset{\sim}{y} = \underset{\sim}{0}$ (SIAM)

Examine $A(\underset{\sim}{x} + \underset{\sim}{y}) \underset{\text{MMDAA}}{=} A\underset{\sim}{x} + A\underset{\sim}{y} = \underset{\sim}{0} + \underset{\sim}{0} = \underset{\sim}{0} \Rightarrow \underset{\sim}{x} + \underset{\sim}{y} \in N(A)$

3) ~~If~~ Suppose $\underset{\sim}{x} \in N(A)$, $\alpha \in \mathbb{C}$. Know $A\underset{\sim}{x} = \underset{\sim}{0}$.

$A(\alpha\underset{\sim}{x}) = \alpha(A\underset{\sim}{x}) = \alpha \cdot \underset{\sim}{0} = \underset{\sim}{0} \Rightarrow \alpha\underset{\sim}{x} \in N(A)$

Ex $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \, 5x_1 - x_2 + 8x_3 = 0 \right\} = N(\begin{bmatrix} 5 & -1 & 8 \end{bmatrix})$ ← a subspace

before

$S \subseteq \mathbb{C}^3$

Theorem V - vector space, $S = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \dots, \underset{\sim}{v_m}\}$ $\langle S \rangle$ is a subspace.

Proof 1) $\underset{\sim}{0} \in \langle S \rangle$?

$$0\underset{\sim}{v_1} + 0\underset{\sim}{v_2} + \dots + 0\underset{\sim}{v_m} \in \langle S \rangle$$

$$\underset{\sim}{0} + \underset{\sim}{0} + \dots + \underset{\sim}{0} \in \langle S \rangle$$

$$\underset{\sim}{0} \in \langle S \rangle$$

2) Suppose $\underset{\sim}{x}, \underset{\sim}{y} \in \langle S \rangle$. There are scalars so that

$$\underset{\sim}{x} = a_1\underset{\sim}{v_1} + a_2\underset{\sim}{v_2} + \dots + a_m\underset{\sim}{v_m}, \quad \underset{\sim}{y} = b_1\underset{\sim}{v_1} + b_2\underset{\sim}{v_2} + \dots + b_m\underset{\sim}{v_m}$$

Consider

$$\underset{\sim}{x}+\underset{\sim}{y} = (a_1\underset{\sim}{v_1} + \dots + a_m\underset{\sim}{v_m}) + (b_1\underset{\sim}{v_1} + \dots + b_m\underset{\sim}{v_m})$$

$$= a_1\underset{\sim}{v_1} + b_1\underset{\sim}{v_1} + \dots + a_m\underset{\sim}{v_m} + b_m\underset{\sim}{v_m}$$ commutativity vector addition

$$= \underbrace{(a_1+b_1)}_{\text{scalar!}}\underset{\sim}{v_1} + (a_2+b_2)\underset{\sim}{v_2} + \dots + (a_m+b_m)\underset{\sim}{v_m}$$ distributivity

So $\underset{\sim}{x}+\underset{\sim}{y}$ is a linear combination of S. So $\underset{\sim}{x}+\underset{\sim}{y} \in \langle S \rangle$

3) HW