1. Use the definition of a linear transformation to verify that $T$ is a linear transformation. (15 points)

$$T: \mathbb{C}^2 \rightarrow \mathbb{C}^2 \quad T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + b \\ a - b \end{pmatrix}$$

2. Find a basis for the range of $T$, $\mathcal{R}(T)$. (20 points)

$$T: \mathbb{C}^3 \rightarrow P_2 \quad T \begin{pmatrix} a \\ b \\ c \end{pmatrix} = (a - 2b - c) + (2a + b + 3c)x + (3a + 3c)x^2$$
3. Consider the linear transformation $S$. (35 points)

$$S: M_{1,3} \rightarrow P_2 \quad S \left( \begin{bmatrix} a & b & c \end{bmatrix} \right) = (2a + 7b + 6c) + (a + 4b + 4c)x + (-b - c)x^2$$

(a) Demonstrate that $S$ is invertible.

(b) Find a formula for $S^{-1}: P_2 \rightarrow M_{1,3}$.

(c) Compute the composition $S^{-1} \circ S$ and explain why this is a partial check on your work above.
4. The linear transformation $R$ is not injective (you may assume this). Find two different vectors, $x$ and $y$, such that $R(x) = R(y)$. (15 points)

$$R: \mathbb{C}^4 \to P_3 \quad R \left( \begin{array}{c} a \\ b \\ c \\ d \end{array} \right) = (b + c - 2d) + (-a + 3c + 5d)x + (2b + 3c - 3d)x^2 + (4b + 8c - 4d)x^3$$

5. Suppose that $T: U \to V$ and $S: U \to V$ are linear transformations which agree on a basis of $U$. That is, for some basis $B = \{u_1, u_2, u_3, \ldots, u_n\}$ of $U$, $T(u_i) = S(u_i)$ for $1 \leq i \leq n$. Prove that $T$ and $S$ are equal functions. (15 points)