Math 290 Name: Dr. Beezer
Exam 4 Spring 2019
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Determine if the set of matrices, T, from the vector space M_{22}, is a linearly independent set. (15 points)

$$T = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} \right\}$$

2. For the matrix A, compute the dimension of the column space and the dimension of the row space. Your answer should illustrate an important theorem from this chapter. State the relevant theorem (in other words, write out the theorem, do not just quote the acronym.) (15 points)

$$A = \begin{bmatrix} 1 & 0 & 2 & -6 & -4 & -3 \\ -1 & 1 & 0 & -2 & 5 & 0 \\ 0 & 0 & 1 & -2 & -1 & -1 \\ 1 & -1 & 3 & -4 & -8 & -3 \end{bmatrix}$$

3. Y is a subspace of P_2, the vector space of polynomials with degree at most 2. (you may assume this much.) Answer the following quesations with *complete* justifications. (40 points)

$Y = \left\{ a + bx + cx^2 \middle| \, a - b + 2c = 0 \right\}$

(a) Prove that the dimension of Y is 2, $\dim(Y) = 2$.

(b) Is $\left\{ 4 + 2x - x^2,\, 5 + 3x - x^2 \right\}$ a basis of Y?

(c) Is $\left\{ 1 + 3x + x^2,\, 2 + 6x + 2x^2 \right\}$ a basis of Y?

(d) Is $\left\{ -5 - x + 2x^2 \right\}$ a basis of Y?

(e) Is $\left\{ -2 + 2x + 2x^2,\, 3 + x - 2x^2 \right\}$ a basis of Y?

4. The set W is a subset of the vector space of column vectors, $\mathbb{C}^3$. Give a careful proof that W is a subspace of $\mathbb{C}^3$. (15 points)

$$W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \, 2a - 3c = 0 \right\}$$

5. Suppose that V is a vector space and that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_m\}$ is a subset of V. Prove that the span of S, $\langle V \rangle$, is a subspace of V. (15 points)