1. For the matrix $A$ below, compute the inverse, or explain how you know $A$ does not have an inverse. (15 points)

$$A = \begin{bmatrix} -1 & 2 & 0 & -2 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & 3 & -4 \\ -1 & 1 & 2 & -5 \end{bmatrix}$$

Answer:

2. A matrix $A$ has the extended echelon form given below. A vector of constants $b$ is given. Decide if the linear system $LS(A, b)$ is consistent or inconsistent, with a careful explanation. Most of the points for this problem will come from the quality of the explanation. (20 points)

$$A = \begin{bmatrix} 1 & 0 & -3 & 0 & -1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 1 & 2 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$
3. Consider the matrix $B$. In each part, find a set of vectors whose span is the column space of $A, C(A)$, and meets the additional requirements, and restrictions on techniques used. Be certain to explain the theorems and definitions employed. (35 points)

$$B = \begin{bmatrix}
-7 & 28 & -11 & 19 & 2 & 12 & -8 \\
11 & -44 & 29 & -65 & -7 & -50 & 32 \\
4 & -16 & 15 & -37 & -4 & -30 & 19 \\
1 & -4 & 16 & -46 & -5 & -40 & 25 \\
0 & 0 & -3 & 9 & 1 & 8 & -5
\end{bmatrix}$$

(a) Use only the definition of a column space.

(b) The set is linearly independent and each vector in the set is a column of $B$.

(c) The set is linearly independent and is constructed using theorems about row spaces in non-trivial ways.

(d) The set is linearly independent and is constructed using Theorem FS ("Four Subspaces") in a non-trivial way.
4. Suppose that \( A \) is an \( m \times n \) matrix. Prove that \((\alpha A)^t = \alpha A^t\). (15 points)

5. Suppose that \( Q \) is an \( n \times n \) unitary matrix and \( x \in \mathbb{C}^n \) is any vector. Prove that \( \|Qx\| = \|x\| \). (This is the statement of a theorem in the book, so do not quote any part of that theorem as part of your answer.) (15 points)