1. Prove that $S$ is a linear transformation. (15 points)

$S : \mathbb{C}^3 \to \mathbb{C}^2$, 

$S \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} b + c \\ a + 2c \end{pmatrix}$

2. For the linear transformation $S$ in the previous problem, compute the preimage $S^{-1} \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. (15 points)
3. Consider the linear transformation $R$ from the vector space of $2 \times 2$ matrices, $M_{22}$, to the vector space of polynomials with largest degree 3, $P_3$. (20 points)

$$R: M_{22} \to P_3, \quad R \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (-2a - b - 5c + d) + (-a - b - 4c)x + (2b + 7c + 3d)x^2 + (a + b + 8c + 4d)x^3$$

(a) Compute the kernel of $R$, $\mathcal{K}(R)$.

(b) Compute the range of $R$, $\mathcal{R}(R)$.

(c) Is $R$ an invertible linear transformation? Why or why not?

4. Consider the invertible linear transformation $T$ from the vector space $\mathbb{C}^3$ to the vector space of polynomials $P_2$. Compute an explicit formula for the inverse of $T$, the linear transformation $T^{-1}: P_2 \to \mathbb{C}^3$. (20 points)

$$T: \mathbb{C}^3 \to P_2, \quad T \left( \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = (-5a - 2b + 2c) + (2a + b - c)x + (-3a - b)x^2$$
5. Suppose that $T: U \to V$ and $S: V \to W$ are linear transformations. Prove that the composition of $S$ and $T$, $S \circ T$, is a linear transformation. (15 points)

6. Suppose that $T: U \to V$ is a linear transformation which has an inverse function, $T^{-1}$. To prove that $T^{-1}$ is a linear transformation would require checking two defining properties. Choose one of the two properties, and prove it. (15 points)