1. Determine if the subset $T$ of the vector space of polynomials with degree at most 3, $P_3$, is linearly independent. (15 points)

$$T = \{ x^3 - 5x^2 + 4x - 2, x^3 - 5x^2 + 3x - 2, -6x^3 + 4x^2 + x + 7 \}$$

2. Does the set $R$ span the vector space of $2 \times 2$ matrices, $M_{22}$? That is, does $\langle R \rangle = M_{22}$? (15 points)

$$R = \left\{ \begin{bmatrix} 4 & 2 \\ 3 & -5 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ 4 & -4 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 2 & -8 \end{bmatrix}, \begin{bmatrix} 7 & 8 \\ 6 & 3 \end{bmatrix} \right\}$$
3. The set $W$ is a subspace of the vector space of polynomials with degree at most 2, $P_2$. (You may assume this.) (40 points)

$$W = \{a + bx + cx^2 \mid a + 2b - c = 0\}$$

(a) Prove that the dimension of $W$ is 2, that is, $\dim(W) = 2$.

(b) Does $K = \{2 + x + 4x^2\}$ span $W$?

(c) Is $L = \{5 + 5x^2, x + 2x^2, 4 + x + 6x^2\}$ a linearly independent subset of $W$?

(d) Is $B = \{-1 + 3x + 5x^2, 3 - 2x - x^2\}$ a basis for $W$?
4. Prove that $U$ is a subspace of the vector space $\mathbb{C}^3$, by using the three-part test of Theorem TSS. (15 points)

$$U = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right| 3a + b + c = 0 \right\}$$

5. Suppose that $\alpha, \beta \in \mathbb{C}$, $V$ is a vector space, $v \in V$, $v \neq 0$, and $\alpha v = \beta v$. Prove that $\alpha = \beta$. (15 points)