Math 290 Name: Dr. Beezer
Exam 4 Spring 2015
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 90 possible points.
You may use Sage to manipulate and row-reduce matrices. Be sure to make it clear what you have input to Sage, and show any output you use to justify your answers.

1. Does the set S span the vector space of 2×3 matrices, M_{23}? (10 points)

$$S=\left\{\begin{bmatrix}1&2&3\\1&0&1\end{bmatrix},\begin{bmatrix}-3&-5&-6\\2&-1&-6\end{bmatrix},\begin{bmatrix}-3&-4&-4\\4&-3&-7\end{bmatrix},\begin{bmatrix}1&-1&-5\\-8&2&4\end{bmatrix},\begin{bmatrix}3&6&8\\5&-5&0\end{bmatrix}\right\}$$

2. Is the set T linearly independent in the vector space of polynomials with degree 2 or less, P_2? (10 points)

$$T=\left\{x^2+3x+3,\ 2x^2+7x+6,\ 2x^2+4x+7\right\}$$

3. Is the set R a basis of the vector space $\mathbb{C}^4$? (10 points)

$$R=\left\{\begin{bmatrix}-1\\1\\1\\2\end{bmatrix},\begin{bmatrix}-3\\2\\5\\2\end{bmatrix},\begin{bmatrix}-4\\3\\5\\2\end{bmatrix},\begin{bmatrix}4\\-4\\-2\\-5\end{bmatrix}\right\}$$

4. Prove that the set $W = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle| \, 3a + 5b = 0 \right\}$ is a subspace of the vector space of column vectors $\mathbb{C}^2$. (15 points)

5. The set $W = \left\{ a + bx + cx^2 \,\middle|\, a + 2b - 3c = 0 \right\}$ is a subspace of the vector space of polynomials in x with degree 2 or less, P_2. (You may assume this.) Determine, with verification, a basis of W. (20 points)

6. Suppose that the set $S = \{\mathbf{u},\, \mathbf{v},\, \mathbf{w}\}$ is a linearly independent subset of the vector space V. Prove that the set $T = \{3\mathbf{u} + 4\mathbf{v} - 8\mathbf{w},\, -\mathbf{u} - \mathbf{v} + 2\mathbf{w},\, 2\mathbf{u} + 2\mathbf{v} - 3\mathbf{w}\}$ is linearly independent in V. (15 points)

7. Suppose that $R = \{\mathbf{v}_1,\, \mathbf{v}_2,\, \ldots,\, \mathbf{v}_m\}$ spans the vector space $\mathbb{C}^n$ and that A is an $n \times n$ nonsingular matrix. Prove that $P = \{A\mathbf{v}_1,\, A\mathbf{v}_2,\, \ldots,\, A\mathbf{v}_m\}$ spans $\mathbb{C}^n$. (10 points)