1. Does the set $S$ span the vector space of $2 \times 3$ matrices, $M_{23}$? (10 points)

$$S = \left\{ \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -3 & -5 & -6 \\ 2 & -1 & -6 \end{bmatrix}, \begin{bmatrix} -3 & -4 & -4 \\ 4 & -3 & -7 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -5 \\ -8 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 6 & 8 \\ 5 & -5 & 0 \end{bmatrix} \right\}$$

2. Is the set $T$ linearly independent in the vector space of polynomials with degree 2 or less, $P_2$? (10 points)

$$T = \{ x^2 + 3x + 3, 2x^2 + 7x + 6, 2x^2 + 4x + 7 \}$$

3. Is the set $R$ a basis of the vector space $\mathbb{C}^4$? (10 points)

$$R = \left\{ \begin{bmatrix} -1 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -2 \\ -5 \end{bmatrix} \right\}$$
4. Prove that the set \( W = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \right| 3a + 5b = 0 \} \) is a subspace of the vector space of column vectors \( \mathbb{C}^2 \). (15 points)

5. The set \( W = \left\{ a + bx + cx^2 \left| a + 2b - 3c = 0 \right. \right\} \) is a subspace of the vector space of polynomials in \( x \) with degree 2 or less, \( P_2 \). (You may assume this.) Determine, with verification, a basis of \( W \). (20 points)
6. Suppose that the set $S = \{u, v, w\}$ is a linearly independent subset of the vector space $V$. Prove that the set $T = \{3u + 4v - 8w, -u - v + 2w, 2u + 2v - 3w\}$ is linearly independent in $V$. (15 points)

7. Suppose that $R = \{v_1, v_2, \ldots, v_m\}$ spans the vector space $\mathbb{C}^n$ and that $A$ is an $n \times n$ nonsingular matrix. Prove that $P = \{Av_1, Av_2, \ldots, Av_m\}$ spans $\mathbb{C}^n$. (10 points)