Math 290 Name: Dr. Beezer
Exam 2 Spring 2015
Chapter V

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to manipulate and row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Is the set R linearly independent or not? (15 points)

$$R = \left\{ \begin{bmatrix} -4 \\ -5 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \\ -1 \\ -4 \end{bmatrix} \right\}$$

2. Is the vector $\mathbf{u}$ in the span of P, $\langle P \rangle$? (15 points)

$$\mathbf{u} = \begin{bmatrix} -11 \\ -6 \\ 5 \\ 10 \end{bmatrix} \qquad P = \left\{ \begin{bmatrix} -3 \\ -2 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 8 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 2 \end{bmatrix} \right\}$$

3. Find a linearly independent set T whose span is the same as the span of S, that is, $\langle T \rangle = \langle S \rangle$. (20 points)

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -5 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -3 \\ 2 \end{bmatrix} \right\}$$

4. Find a linearly independent set of vectors T that span the null space of A, that is, $\langle T \rangle = \mathcal{N}(A)$. (20 points)

$$A = \begin{bmatrix} -1 & 0 & -4 & 2 & 6 & -2 & 0 \\ 1 & -1 & -1 & -3 & -5 & 2 & -1 \\ 0 & -1 & -5 & 0 & 6 & 0 & 1 \\ 1 & -2 & -6 & -3 & 1 & 3 & 1 \end{bmatrix}$$

5. Construct a careful proof, using Definition CVE (Column Vector Equality), of the following: if $\mathbf{v} \in \mathbb{C}^m$, then $0\mathbf{v} = \mathbf{0}$. Be sure to provide justification for each step of your proof. (15 points)

6. Prove that the inner product is anti-commutative. That is, for $\mathbf{u}, \mathbf{w} \in \mathbb{C}^n$, $\langle \mathbf{u}, \mathbf{w} \rangle = \overline{\langle \mathbf{w}, \mathbf{u} \rangle}$. (15 points)