1. Consider the linear transformation $T$ below. $P_1$ is the vector space of polynomials with degree at most 1, and $M_{22}$ is the vector space of $2 \times 2$ matrices. (45 points)

$$T: M_{22} \to P_1, \quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a + 2b + 3c - 3d) + (3a - b + 2c + 5d)x$$

(a) Compute the kernel of $T$, $\mathcal{K}(T)$.

(b) Is $T$ injective? Why? If not, find two elements of the domain that $T$ takes to the same element of the codomain.

(c) Compute the range of $T$, $\mathcal{R}(T)$.

(d) Is $T$ surjective? Why? If not, find an element of the codomain with an empty preimage.

(e) If $T$ invertible? Why?
2. Consider the linear transformation $S$ below, which is invertible (you may assume this). Find a formula for the outputs of the inverse linear transformation $S^{-1}$. $P_1$ is the vector space of polynomials with degree at most 1. (25 points)

$S: P_1 \rightarrow \mathbb{C}^2, \quad S(a + bx) = \begin{bmatrix} 2a + 5b \\ a + 3b \end{bmatrix}$
3. Prove that the function $T$ below is a linear transformation. $P_1$ is the vector space of polynomials with degree at most 1. (15 points)

$$T: P_1 \rightarrow \mathbb{C}^2, \quad T(a + bx) = \begin{bmatrix} 3a - b \\ 4b \end{bmatrix}$$

4. Suppose that $S: U \rightarrow V$ is an invertible linear transformation. Then prove that $S^{-1}$ has one of the two defining properties of a linear transformation (either property, your choice, for full credit). (15 points)