Math 290 Name: Dr. Beezer
Exam 6 Spring 2014
Chapter LT

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices and include the output in your answer as justification.

1. Consider the linear transformation T below. P_1 is the vector space of polynomials with degree at most 1, and M_{22} is the vector space of 2×2 matrices. (45 points)

   $$T\colon M_{22} \to P_1, \quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a + 2b + 3c - 3d) + (3a - b + 2c + 5d)x$$

   (a) Compute the kernel of T, $\mathcal{K}(T)$.

   (b) Is T injective? Why? If not, find two elements of the domain that T takes to the same element of the codomain.

   (c) Compute the range of T, $\mathcal{R}(T)$.

   (d) Is T surjective? Why? If not, find an element of the codomain with an empty preimage.

   (e) If T invertible? Why?

2. Consider the linear transformation S below, which is invertible (you may assume this). Find a formula for the outputs of the inverse linear transformation S^{-1}. P_1 is the vector space of polynomials with degree at most 1. (25 points)

$$S\colon P_1 \to \mathbb{C}^2, \quad S\left(a + bx\right) = \begin{bmatrix} 2a + 5b \\ a + 3b \end{bmatrix}$$

3. Prove that the function T below is a linear transformation. P_1 is the vector space of polynomials with degree at most 1. (15 points)

   $$T\colon P_1 \to \mathbb{C}^2, \quad T(a+bx) = \begin{bmatrix} 3a-b \\ 4b \end{bmatrix}$$

4. Suppose that $S\colon U \to V$ is an invertible linear transformation. Then prove that S^{-1} has one of the two defining properties of a linear transformation (either property, your choice, for full credit). (15 points)