Math 290 Name: Dr. Beezer
Exam 3 Spring 2014
Chapter M

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use reduced row-echelon from Sage as justification for parts of your answers, so long as you explain your input and list the output in your solution.

1. Solve the following system of equations *using an inverse of a matrix.* No credit will be given for solutions obtained using a different method. (15 points)

$$\begin{aligned} x_1 - 4\,x_2 + 3\,x_3 &= -5 \\ -x_1 + 5\,x_2 - 5\,x_3 &= 3 \\ -x_1 + 4\,x_2 - 2\,x_3 &= 9 \end{aligned}$$

2. Compute the inverse of the matrix A. (10 points)

$$A = \begin{bmatrix} -1 & 0 & 6 & -4 \\ -1 & -2 & 1 & 1 \\ -2 & -3 & 5 & -1 \\ 2 & 2 & -3 & -1 \end{bmatrix}$$

3. Compute the requested versions of row and column spaces for the matrix B. (50 points)

$$B = \begin{bmatrix} 2 & -5 & 9 & 33 & 1 & -8 \\ -3 & 24 & -63 & -132 & -11 & 38 \\ 1 & -7 & 18 & 39 & 3 & -11 \\ 0 & -7 & 21 & 35 & 4 & -11 \\ 0 & -2 & 6 & 10 & 1 & -3 \end{bmatrix}$$

(a) The column space of B, using only the definition.

(b) The column space of B, as the span of a linearly independent set containing only columns of B.

(c) The column space of B, as the span of a linearly independent set obtained from the row space of a different matrix.

(d) The column space of B, as the span of a linearly independent set derived from the extended echelon form of B.

(e) The row space of B, as the span of a linearly independent set.

4. Suppose that A and B are $m \times n$ matrices. Prove that $A + B = B + A$, giving a reason for each step of your proof. (10 points)

5. Suppose A is an $m \times n$ matrix and B, C are $n \times p$ matrices. Prove that $A(B + C) = AB + AC$. (This is Theorem MMDAA, so do more than just quote this result.) (15 points)