Chapter V

Show all of your work and explain your answers fully. There is a total of 100 possible points. You may use Sage to create and row-reduce matrices.

1. Determine if the set of column vectors, $T$, is linearly independent or not, including an accurate justification for your answer. (15 points)

$$T = \left\{ \begin{bmatrix} -1 \\ -3 \\ 0 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 4 \\ 3 \\ -2 \end{bmatrix} \right\}$$

2. Determine if the vector $w$ is in the set $U = \langle T \rangle$. (15 points)

$$w = \begin{bmatrix} 2 \\ -2 \\ -4 \\ -1 \end{bmatrix} \quad T = \left\{ \begin{bmatrix} 1 \\ -4 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 3 \\ -1 \end{bmatrix} \right\}$$
3. Find a linearly independent set \( R \) whose span is the null space of \( A \) (in other words, \( \langle R \rangle = N(A) \)). Explain how you know your answer has the required properties. (20 points)

\[
A = \begin{bmatrix}
-3 & 1 & -5 & 5 & 3 \\
-4 & 1 & -7 & 6 & 5 \\
-4 & 1 & -7 & 6 & 5
\end{bmatrix}
\]

4. Find a linearly independent set \( T \) with the same span as \( S \) (in other words \( \langle T \rangle = \langle S \rangle \)). (20 points)

\[
S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -5 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} \right\}
\]
5. Suppose that \( u \in \mathbb{C}^n \) is a vector. Prove that \( 1u = u \). Provide reasons for each deduction and employ our indexing notation for entries of vectors. (Do not simply quote this as a result from Theorem VSPCV.) (15 points)

6. Suppose that \( A = [A_1 | A_2 | A_3 | \ldots | A_n] \) is a matrix and that both \( x, y \in \mathbb{C}^n \) are solutions to \( \mathcal{L}S(A, b) \). Prove that \( x - y \) is a solution to the homogeneous system \( \mathcal{L}S(A, 0) \). (You may assume that vector subtraction is defined by \( [x - y]_i = [x]_i - [y]_i \), for \( 1 \leq i \leq n \).) (15 points)