Math 290 Name: Dr. Beezer
Exam 2 Spring 2014
Chapter V

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to create and row-reduce matrices.

1. Determine if the set of column vectors, T, is linearly independent or not, including an accurate justification for your answer. (15 points)

$$T = \left\{ \begin{bmatrix} -1 \\ -3 \\ 0 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 4 \\ 3 \\ -2 \end{bmatrix} \right\}$$

2. Determine if the vector $\mathbf{w}$ is in the set $U = \langle T \rangle$. (15 points)

$$\mathbf{w} = \begin{bmatrix} 2 \\ -2 \\ -4 \\ -1 \end{bmatrix} \qquad T = \left\{ \begin{bmatrix} 1 \\ -4 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 3 \\ -1 \end{bmatrix} \right\}$$

3. Find a linearly independent set R whose span is the null space of A (in other words, $\langle R\rangle = \mathcal{N}(A)$). Explain how you know your answer has the required properties. (20 points)

$$A = \begin{bmatrix} -3 & 1 & -5 & 5 & 3 \\ -4 & 1 & -7 & 6 & 5 \\ -4 & 1 & -7 & 6 & 5 \end{bmatrix}$$

4. Find a linearly independent set T with the same span as S (in other words $\langle T\rangle = \langle S\rangle$). (20 points)

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -5 \\ -3 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -5 \\ 0 \end{bmatrix} \right\}$$

5. Suppose that $\mathbf{u} \in \mathbb{C}^n$ is a vector. Prove that $1\mathbf{u} = \mathbf{u}$. Provide reasons for each deduction and employ our indexing notation for entries of vectors. (Do not simply quote this as a result from Theorem VSPCV.) (15 points)

6. Suppose that $A = [\mathbf{A}_1|\mathbf{A}_2|\mathbf{A}_3|\ldots|\mathbf{A}_n]$ is a matrix and that both $\mathbf{x}, \mathbf{y} \in \mathbb{C}^n$ are solutions to $\mathcal{LS}(A,\, b)$. Prove that $\mathbf{x} - \mathbf{y}$ is a solution to the homgeneous system $\mathcal{LS}(A,\, \mathbf{0})$. (You may assume that vector subtraction is defined by $[\mathbf{x} - \mathbf{y}]_i = [\mathbf{x}]_i - [\mathbf{y}]_i$, for $1 \le i \le n$.) (15 points)