

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.

You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers.

1. Solve the following system of linear equations and express the solutions as a set of column vectors. (15 points)

$$-2x_1 + 3x_2 - 2x_3 + x_4 = 1$$

$$-x_1 + x_2 - x_3 - x_4 = 0$$

$$3x_1 - 3x_2 + 4x_3 + 6x_4 = 1$$

$$3x_2 - 2x_3 + 4x_4 = 5$$

2. Solve the following system of linear equations and express the solutions as a set of column vectors. (20 points)

$$x_1 + 2x_2 + 7x_3 + 6x_4 - 3x_5 = 12$$

$$x_1 - 3x_3 - x_4 = -1$$

$$-x_2 - 5x_3 - 4x_4 + 2x_5 = -8$$



3. Without using Sage, find a matrix  $B$  in reduced row-echelon form which is row-equivalent to  $A$ . It is especially important to show all of your work, so it is clear you have not used Sage. (20 points)

$$A = \begin{bmatrix} 1 & -2 & -4 & -6 \\ 2 & -3 & -5 & -11 \\ 1 & 2 & 8 & -2 \end{bmatrix}$$

4. Determine if the matrix below is nonsingular or singular. Explain your reasoning carefully and thoroughly. (15 points)

$$\begin{bmatrix} 1 & 1 & 0 & 2 & 2 & -4 & 5 \\ -1 & 0 & 1 & 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 1 & 2 & -7 & 6 \\ 1 & 1 & -1 & 2 & 2 & -1 & 1 \\ -1 & -2 & 0 & -5 & -3 & 4 & -7 \\ 2 & 1 & 0 & 1 & 3 & -7 & 6 \\ 1 & 1 & -2 & 0 & 1 & 4 & -3 \end{bmatrix}$$



5. Compute the null space of the matrix  $D$ . (15 points)

$$D = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -2 & -3 & 3 & 4 \end{bmatrix}$$

6. Suppose that  $A$  is the augmented matrix of a system of equations with  $n$  variables. Suppose that  $B$  is a matrix that is row-equivalent to  $A$  and is in reduced row-echelon form, with  $r = n + 1$ . Give a careful, well-written, proof that the system of equations is inconsistent. (15 points)

