Math 290 Name: Dr. Beezer
Exam 6 Fall 2013
Chapter LT

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
As justification for your work, you may use Sage routines designed for matrices, but not routines designed for linear transformations.

1. Prove that the following function is a linear transformation. (15 points)

$$T\colon \mathbb{C}^2 \to \mathbb{C}^2, \quad T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a + 2b \\ a - b \end{bmatrix}$$

2. The linear transformation S is invertible (you may assume this). Find a formula for values of the inverse linear transformation, S^{-1}. (15 points)

$$S\colon P_2 \to \mathbb{C}^3, \quad S\left(a + bx + cx^2\right) = \begin{bmatrix} -2a + b + c \\ -3a + b - c \\ -2a - 3c \end{bmatrix}$$

3. In the following questions, analyze the linear transformation T. (40 points)

$$T\colon M_{22} \to P_2, \quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (-2a - 2b - 4c + 6d) + (b + c - 2d)x + (a + c - d)x^2$$

(a) Find a basis for the kernel of T, $\mathcal{K}(T)$.

(b) Is T injective? Why or why not?

(c) Find a basis for the range of T, $\mathcal{R}(T)$.

(d) Is T surjective? Why or why not?

4. Suppose that $T\colon U \to V$ is a linear transformation. Prove Theorem LTTZZ: $T(\mathbf{0}) = \mathbf{0}$. Give a clear proof using words (not arrows) to indicate implications and giving a reason for every step. This will be graded partly on good style — a bare sequence of equations with no structure is unlikely to get full credit. (15 points)

5. Suppose that $T\colon U \to V$ and $S\colon V \to W$ are linear transformations. Prove the following relationship between ranges and composition: $\mathcal{R}(S \circ T) \subseteq \mathcal{R}(S)$. (15 points)