Math 290 | Name: | Dr. Beezer

Exam 4 | Fall 2013

Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices and to multiply matrices, so long as you explain your input and show your output in your work. No other use of Sage may be used as justification for your answers.

1. Determine if the following set, T, as a subset of the vector space $M_{2,3}$, is linearly independent or not. (15 points)

$$T = \left\{ \begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \\ 2 & -4 & -1 \end{bmatrix}, \begin{bmatrix} -5 & 4 & -3 \\ 2 & -8 & 2 \end{bmatrix} \right\}$$

2. Prove that U is a subspace of P_2, the vector space of polynomials of degree 3 or less. (15 points)

$$U = \left\{ a + bx + cx^2 \,\middle|\, 3a + 6b - 3c = 0 \right\}.$$

3. U is identical to the subspace from the previous question. Show that the dimension of U is 2, by exhibiting a basis for U, being certain to establish carefully that the basis is both a spanning set for U and is linearly independent. (15 points)

   $U = \left\{a + bx + cx^2 \middle| \, 3a + 6b - 3c = 0\right\}$

4. U is identical to the subspace from the previous questions. Now assume that you know the dimension of U is 2. Answer the following with careful explanations of your answer. (25 points)

   $U = \left\{a + bx + cx^2 \middle| \, 3a + 6b - 3c = 0\right\}$

   (a) Does the set $R = \left\{4 - x + 2x^2\right\}$ span U?

   (b) Is the set $S = \left\{-1 + 2x + 3x^2,\, 3 - x + x^2,\, -8 + 3x - 2x^2\right\} \subseteq U$ linearly independent?

   (c) Is the set $T = \left\{-5 + 3x + x^2,\, -8 + 5x + 2x^2\right\}$ a basis of U?

5. Suppose that $S = \{\mathbf{u},\, \mathbf{v},\, \mathbf{w}\}$ is a linearly independent subset of a vector space V. Prove that T is also a linearly independent subset of V. (15 points)

   $T = \{\mathbf{u} - \mathbf{v} + \mathbf{w},\, \mathbf{u} + \mathbf{v} - 4\mathbf{w},\, 2\mathbf{u} + \mathbf{v} - 6\mathbf{w}\}$

6. Suppose that $\alpha \in \mathbb{C}$, V is a vector space, $\mathbf{v} \in V$ and $\alpha\mathbf{v} = \mathbf{0}$. Prove that either $\alpha = 0$ or $\mathbf{v} = \mathbf{0}$. (Notice that this is Theorem SMEZV so you are being asked to do more than just quote the theorem.) (15 points)