Math 290 Name: Dr. Beezer
Exam 3 Fall 2013
Chapter M

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
You may use Sage to row-reduce matrices and to multiply matrices, so long as you explain your input and show your output in your work. No other use of Sage may be used as justification for your answers.

1. Compute the inverse of the matrix A below, if it exists. You may use Sage, subject to the restrictions above. (15 points)

$$A = \begin{bmatrix} 5 & -7 & 7 & 4 \\ 7 & -8 & 9 & 5 \\ 3 & -4 & 4 & 3 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

2. The coefficient matrix of the system below is the matrix A from the previous problem. *Use your answer to the previous question* to
(a) *first*, determine how many solutions the system has, and
(b) *second*, determine the solution set of the system.
Label parts (a) and (b) clearly in your answer, answer (a) without reference to (b), and be certain to base your answers on the results of the previous problem. There will be no partial credit for answers obtained by other approaches. (15 points)

$$\begin{aligned} 5x_1 - 7x_2 + 7x_3 + 4x_4 &= 16 \\ 7x_1 - 8x_2 + 9x_3 + 5x_4 &= 20 \\ 3x_1 - 4x_2 + 4x_3 + 3x_4 &= 13 \\ x_1 - x_2 + x_3 + x_4 &= 5 \end{aligned}$$

3. Answer the following questions about the matrix B below. (40 points)

$$B = \begin{bmatrix} 7 & 12 & -18 & -61 & 27 \\ 4 & 2 & -5 & -15 & 11 \\ -1 & -3 & 4 & 14 & -5 \\ 0 & -1 & 1 & 4 & -1 \end{bmatrix}$$

(a) For the column space of B, find a linearly independent set S, with $\mathcal{C}(B) = \langle S \rangle$ and S contains only columns of B.

(b) For the row space of B, find a linearly independent set T, with $\mathcal{R}(B) = \langle T \rangle$.

(c) For the column space of B, *use a technique substantially different from part (a)* to find a new linearly independent set R, with $\mathcal{C}(B) = \langle R \rangle$.

(d) Find a nonzero vector $\mathbf{c}$ so that the system $\mathcal{LS}(B, \mathbf{c})$ is consistent. Give an explanation of how you know the system is consistent *without simply solving the system.*

(e) Find a vector $\mathbf{d}$ so that the system $\mathcal{LS}(B, \mathbf{d})$ is inconsistent. Give an explanation of how you know the system is inconsistent *without simply attempting to solve the system.*

4. Suppose that $\alpha \in \mathbb{C}$ and $A, B \in M_{mn}$. Give a careful proof that $\alpha\,(A + B) = \alpha A + \alpha B$. (Notice that this is Propery DMAM so you are being asked to do more than just quote the statement of the property.) (15 points)

5. Suppose A is an $m \times n$ matrix and B and C are $n \times p$ matrices. Prove that $A(B+C) = AB+AC$. (Notice that this is Theorem MMDAA so you are being asked to do more than just quote the statement of the theorem.) (15 points)