Math 290 | Name: | Dr. Beezer
Exam 3 | | Spring 2012
Chapter M

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Use Sage only to row-reduce matrices or to compute extended echelon form, except where indicated in a problem statement. Include the results of these computations in your answers and describe the input used.

1. Other than actually asking Sage to compute the inverse of the matrix A below, what Sage command could you use to see that A has an inverse? Then compute the inverse of A only using just the reduced row-echelon form command, `.rref()`, on an appropriate matrix. (You can create the "appropriate" matrix with any Sage commands you like.) (15 points)

$$A = \begin{bmatrix} -2 & -3 & -1 & 0 \\ 3 & 4 & 2 & -2 \\ -3 & -4 & -1 & 0 \\ -4 & -6 & -4 & 5 \end{bmatrix}$$

2. Determine a linearly independent spanning set for the column space of B in two different ways, meeting the requirements given. (20 points)

$$B = \begin{bmatrix} 0 & -1 & -1 & 2 & -1 \\ -2 & 7 & 3 & -16 & 13 \\ 3 & -10 & -4 & 23 & -19 \\ -3 & 10 & 4 & -23 & 19 \end{bmatrix}$$

(a) The set contains only vectors that are columns of B.

(b) The set should be obtained in the most computationally efficient manner possible.

3. Consider the matrix B from the previous question. (20 points)

   (a) Use the matrix L from the extended echelon form to compute a linearly independent set whose span equals the column space of B.

   (b) Use the matrix L from the extended echelon form to compute a linearly independent set whose span equals the left null space of B.

4. Suppose that A and B are both $m \times n$ matrices. Prove that $(A+B)^t = A^t + B^t$. (15 points)

5. When computing the extended echelon form of an $m \times n$ matrix A we compute $[A|I_m] \xrightarrow{\text{RREF}} [B|J]$. Prove that J is nonsingular. (15 points)

6. Suppose $\alpha \in \mathbb{C}$ is a scalar, A is an $m \times n$ matrix and B is an $n \times p$ matrix. Prove that $(\alpha A)\, B = A\,(\alpha B)$. (15 points)