

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Use Sage only to row-reduce matrices, and not at all for the one question where Sage is banned.

1. Find the solution set for the following system of linear equations. (15 points)

$$2x_1 + 5x_2 - 3x_3 + 4x_4 = 10$$

$$7x_1 + x_2 + 7x_3 + 2x_4 = -10$$

$$6x_1 - x_2 - x_3 - 2x_4 = 18$$

$$-x_1 + 3x_2 + 8x_3 + 5x_4 = -33$$

2. Find the solution set for the following system of linear equations. (15 points)

$$3x_1 + x_2 + 4x_3 + 3x_4 - x_5 = 0$$

$$-x_2 + 3x_3 + 8x_4 - 3x_5 = 2$$

$$-x_1 - x_2 + x_3 + 5x_4 - 2x_5 = 1$$

$$4x_1 + x_2 + 5x_3 + 4x_4 - x_5 = 2$$



3. Without using Sage, find a matrix B that is reduced row-echelon form and is row-equivalent to A . (15 points)

$$A = \begin{bmatrix} 1 & -2 & -5 & -3 \\ 0 & 1 & 0 & -3 \\ -1 & 0 & 4 & 7 \end{bmatrix}$$

4. Find the null space of the matrix F , $\mathcal{N}(F)$. (15 points)

$$F = \begin{bmatrix} 0 & -1 & 4 & 7 \\ -3 & 4 & -7 & -4 \\ -4 & 4 & -3 & 6 \\ -1 & 0 & 3 & 8 \end{bmatrix}$$



5. Determine if the matrix C is singular or nonsingular. (15 points)

$$C = \begin{bmatrix} -2 & -1 & -4 & 5 \\ 3 & 2 & 8 & -6 \\ -3 & -2 & -7 & 8 \\ 3 & 1 & 6 & -6 \end{bmatrix}$$

6. Suppose that two systems of linear equations are equivalent. Prove that if one system is homogeneous, then the other must also be homogeneous. (15 points)

