Reading Questions, Chapter 01

1. Use Sage to express 123456792 as a product of prime numbers.
2. Find the greatest common divisor of 84 and 52.
3. Find integers r and s so that $r(84) + s(52) = \gcd(84, 52)$.
4. Explain the use of the term "induction hypothesis."
5. What is Goldbach's Conjecture? And why is it called a "conjecture"?

Reading Questions, Chapter 02

1. In the group $\mathbb{Z}_8$ compute (a) $6 + 7$, (b) 2^{-1}
2. In the group $U(16)$ compute (a) $5 \cdot 7$, (b) 3^{-1}
3. State the definition of a group.
4. Explain a single method that will decide if a subset of a group is itself a subgroup.
5. Explain the origin of the term "abelian" for a commutative group.

Reading Questions, Chapter 03

1. What is the order of the element 3 in U(20)?
2. What is the order of the element 5 in U(23)?
3. Find three generators of $\mathbb{Z}_8$.
4. Find three generators of the 5^{th} roots of unity.
5. Show how to compute $15^{40} \pmod{23}$ efficiently by hand. Check your answer with SAGE.

Reading Questions, Chapter 04

1. Express $(1\,3\,4)(3\,5\,4)$ as a cycle, or a product of disjoint cycles.
2. What is a transposition?
3. What does it mean for a permutation to be even or odd?
4. Describe another group that is fundamentally the same as A_3.
5. Write the elements of the symmetry group of a pentagon using permutations in cycle notation.

Reading Questions, Chapter 05

1. State Lagrange's Theorem in your own words.
2. Determine the left cosets of $\langle 3 \rangle$ in $\mathbb{Z}_9$.
3. The set $\{(), (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)\}$ is a subgroup of S_4. What is its index in S_4?
4. Suppose G is a group of order 29. Describe G.
5. $p = 137909$ is a prime. Explain how to compute $57^{137909} \pmod{137909}$ without a calculator.

Reading Questions, Chapter 08

1. Determine the order of $(1, 2)$ in $\mathbb{Z}_4 \times \mathbb{Z}_8$.
2. List three properties of a group that are preserved by an isomorphism.
3. Find a group isomorphic to $\mathbb{Z}_{15}$ that is an external direct product of two non-trivial subgroups.
4. Explain why we can now say "*the* infinite cyclic group"?
5. Compare and contrast external direct products and internal direct products.

Reading Questions, Chapter 09a

1. Let G be the group of symmetries of an equilateral triangle, expressed as permutations of the vertices numbered $1, 2, 3$. Let H be the subgroup $H = \{(1\ 2)\}$. Build the left and right cosets of H in G.
2. Based on your answer to the previous question, is H normal in G? Explain why or why not.
3. $8\mathbb{Z}$ is a normal subgroup in $\mathbb{Z}$. In the factor group $\mathbb{Z}/8\mathbb{Z}$ perform the computation $(3 + 8\mathbb{Z}) + (7 + 8\mathbb{Z})$.
4. List two statements about a group G and a subgroup H that are equivalent to "H is normal in G."
5. In your own words, what is a factor group?

Reading Questions, Chapter 09b

1. Consider the function $\phi : \mathbb{Z}_{10} \to \mathbb{Z}_{10}$ defined by $\phi(x) = x + x$. Prove that ϕ is a group homomorphism.
2. For ϕ defined in the previous question, explain why ϕ is not a group isomorphism..
3. Compare and contrast isomorphisms and homomorphisms.
4. State the definition of a simple group. What is interesting about simple groups historically?
5. "For every normal subgroup there is a homomorphism, and for every homomorphism there is a normal subgroup." Explain the (precise) basis for this (vague) statement.

Reading Questions, Chapter 11

1. How many abelian groups are there of order $200 = 2^3 5^2$?
2. How many abelian groups are there of order $729 = 3^6$?
3. Find a subgroup of order 6 in $\mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_3$.
4. It can be shown that an abelian group of order 72 contains a subgroup of order 8. What are the possibilities for this subgroup?
5. What is a principal series of the group G?

Reading Questions, Chapter 12

1. Give an informal description of a group action.
2. Describe the class equation.
3. What are the groups of order 49?
4. How many switching fuctions are there with 5 inputs?
5. The "Historical Note" mentions the proof of Burnside's Conjecture. How long was the proof?

Reading Questions, Chapter 13

1. State Sylow's First Theorem.
2. How many groups are there of order 69? Why?
3. Give two descriptions, different in character, of the normalizer of a subgroup.
4. What's all the fuss about Sylow's Theorems?
5. Name one of Sylow's academic great-great-great-great-great-great-grandchildren.
   (That's (great-)^{6}grand-children.)