For questions requesting a computation (numerical or symbolic) report just the answer, without intermediate computations. For a question with a yes/no answer, provide an explanation of your answer.

Section 5.1

1. Find an approximation to the area under $f(x) = x^2$ on the interval $[3, 9]$ by using the areas of three rectangles and the midpoint rule.
2. Explain why using a sum of areas of rectangles will only yield an *approximation* to the true area under a curve.
3. Which would you expect to provide a better approximation to the area under a curve: 10 rectangles using the midpoint rule, or 5,000 rectangles using a lower sum?

Section 5.2

1. Compute the value of $\displaystyle\sum_{k=3}^{5} k^2 - 2k$.
2. What is a partition?
3. Use the textbook's web site (link on course page) to read Bernhard Riemann's biography. What disease afflicted Riemann late in his life?

Section 5.3

1. If $\displaystyle\int_2^6 g(x)\,dx = 12.7$, what is the value of $\displaystyle\int_6^2 g(x)\,dx$?
2. What property of a function will *guarantee* that the function is integrable?
3. Give an example of a function that is not integrable (and that is fundamentally different from Example 5.3.1).

Section 5.4

1. Why is the Fundamental Theorem of Calculus such a big deal?
2. Compare and contrast the two versions of the Fundamental Theorem of Calculus (Theorem 4, Parts 1 and 2).
3. Compute, exactly, the area under $f(x) = 3x^2 + 4x$ above the interval $[2, 4]$ by using an antiderivative.

Section 5.5

1. Subsitution is a technique for indefinite integration (aka “anti-differentiation”). It has its genesis in what rule for differentiation?
2. Find an antiderivative of $h(x) = 2x\cos(x^2)$.
3. Find all of the antiderivatives of $\ell(x) = x^3(x^4 + 17)^8$.

Section 5.6

1. When using substitution on a definite integral, do you prefer to change the limits of integration immediately, or replace the substitution later, just before evaluation of the integral? Why?
2. Find the area between the curves described by $y = x^2$ and $y = 4x - 4$ and above the interval $[2, 3]$.
3. Use an integral with respect to y to find the area between the curves described by the equations $x = y^2$ and $x - 2y = 0$.

Section 5.7

1. What is unusual about how the natural logarithm is defined in this section?
2. Why is the result negative when computing the natural logarithm of a number between 0 and 1?
3. Where does the number e come from? What is its value? Approximately, or exactly?

Section Strang 1.1, 1.2

1. If you have the graph of position, f, how do you create the graph of velocity, v?
2. If you have the graph of velocity, v, how do you create the graph of position, f?
3. What is the central idea of Section 1.2? Do you recognize it?

Section Strang 1.3, 1.4

1. When velocity increases linearly, how does position increase?
2. Comment on the “Calculus and the Law” example on page 17. If your conviction for speeding was upheld by the judge, would you appeal?
3. Suppose you have oscillatory motion where velocity at time t is given by $v(t) = 4\sin(3t)$. What is the change in position of the object between times $t = \pi$ and $t = 2\pi$?

Section 6.1

1. Take the area bounded by the x-axis, the curve $y = x^2$ and $x = 2$ and rotate it around the x-axis to create a solid of revolution. What volume results?

2. Take the area bounded by the y-axis, the curve $y = x^2$ and $y = 4$ and rotate it around the y-axis to create a solid of revolution. What volume results?

3. What is Cavalieri's Principle?

Section 6.2

1. Take the area bounded by the x-axis, the curve $y = x^2$ and $x = 2$ and rotate it around the y-axis to create a solid of revolution. What volume results when you use cylindrical shells to set up the definite integral?

2. Take the area bounded by the y-axis, the curve $y = x^2$ and $y = 4$ and rotate it around the y-axis to create a solid of revolution. What volume results when you use cylindrical shells to set up the definite integral?

3. Why have two different methods for computing the volume of a solid of revolution?

Section 6.3

1. Compute the length of $y = 2x + 3$ between the points $(2, 7)$ and $(5, 13)$. Report both the differential ds you use and the numerical value that results for the arc length.

2. Explain how you can check the answer to the previous question without using calculus, perform the check, and then comment on the result.

3. Explain why it is natural for the derivative of $y = f(x)$ to enter into the formula for the length of a curve.

Section 6.4

1. Rotate the segment of the curve $y = 2x + 3$ between the points $x = 2$ and $x = 5$, around the x-axis. Compute the resulting surface area.

2. Rotate the segment of the curve $y = 2x + 3$ between the points $y = 7$ and $y = 13$, around the y-axis. Compute the resulting surface area.

3. How is the computation of the surface area of a solid of revolution similar to the computation of arc length?

Section 6.5

1. Describe the basic characteristics of how a quantity changes that will then lead to exponential growth.
2. Suppose the balance in your bank account doubles every 18 years. How much bigger will your bank account be 54 years from now?
3. Suppose the balance in your bank account doubles every 18 years. How long until your bank account is three times what it is now?

Section 6.6

1. Why does the computation of work require a definite integral?
2. A 2 kg weight is suspended from a spring. This causes the spring to stretch 12 cm. What is the value of k, the spring constant from Hooke's Law?
3. As the spring in the previous question was stretched from its natural length to a length 12 cm greater, how much work was done? Express your answer in units using meters.

Section 6.7

1. Why is it necessary to use calculus to find a center of mass? In other words, why isn't this just a geometry problem?
2. What is the difference between a centroid and a center of mass?
3. Describe the location of the centroid of a triangle. (Hint: read through the exercises.)

Section 7.1

1. Use integration by parts to find the indefinite integral, $\int x \sin(x)\, dx$.
2. Compute the definite integral $\int_0^{5\pi} x \sin(x)\, dx$
3. Which derivative rule is the origin of the integration-by-parts technique?

Section 7.2

1. $\int \sin^3(x)\, dx$
2. $\int \sqrt{1+\cos(2x)}\, dx$
3. $\int \cos(2x)\cos(5x)\, dx$

Section 7.3

1. If an integrand contained the expression $(x^2 + 16)^{15}$, what trigonometric substitution might you try?

2. If an integrand contained the expression $2x^2 - 50$, what trigonometric substitution might you try?

3. In the context of this section, what is a reference triangle?

Section 7.4

1. Suppose you have an integrand that is a 7th degree polynomial divided by a 4th degree polynomial. What is your first step in computing this integral?

2. Using partial fractions, what is different about handling an irreducible quadratic factor versus a linear factor from the denominator?

3. After expanding an integrand via partial fractions, what types of integrals remain?

Section 7.5

1. What is integral formula #77 in the table in your textbook?

2. What is a CAS?

3. In the context of an integral table, what is a reduction formula?

Section 7.6

1. Approximate the area under $f(x) = x^3$ between $x = 2$ and $x = 10$ using the trapezoidal rule with $n = 4$.

2. What is the fundamental difference between the trapezoidal rule and Simpson's rule?

3. Why does this section include such a careful discussion of error bounds for the approximations obtained with the trapezoidal rule and Simpson's rule?

Section 7.7

1. Describe, in words, the two basic ways an integral can be improper.

2. We cannot apply the Fundamental Theorem of Calculus directly to improper integrals of Type II. Why not?

3. What value does $\displaystyle\int_3^\infty \frac{1}{x^5}$ converge to?

Section 8.1

1. What are the first four terms of the sequence given by $a_n = n^2 - n$?

2. Determine the fifth term in the sequence defined recursively by $a_1 = 2$, $a_n = 3a_n - 2$.

3. What is the limit of the sequence given by $a_n = \dfrac{3n^2 - 5}{6n^2 + 4}$?

Section 8.2

1. What is the difference between a series and a sequence?

2. What is the value of $\displaystyle\sum_{i=0}^{\infty} \left(\frac{1}{5}\right)^i$?

3. What is a "telescoping" series?

Section 8.3

1. Why is the conclusion of the integral test so much better than the conclusion of the n-th term test?

2. Why would we say the harmonic series is right on the border between convergence and divergence?

3. According to the integral test, the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges. What does it converge to?

Section 8.4

1. What is the difference in the *conclusions* of the comparison test versus the limit comparison test?

2. Explain why the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + 18}$ converges.

3. The comparison tests require us to compare a series to a series we already understand. Which broad classes of series to we understand fully with regard to their convergence?

Section 8.5

1. What happens when you apply the ratio test to a geometric series?

2. What happens when you apply the root test to a geometric series?

3. What happens when you apply the ratio test to the harmonic series?

Section 8.6

1. What is different about the series we are studying in this section?

2. Why does the alternating harmonic series, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, converge, when the harmonic series does not?

3. Give an example of an alternating series that does not converge.

Section 8.7

1. What is a power series?
2. Given a power series, what are the two most natural questions to ask about it?
3. What happens when you differentiate a power series term-by-term?

Section 8.8

1. Describe the three general steps required to construct a Taylor series.
2. What is the difference between a Taylor series and a Maclaurin series?
3. What happens when you form a Taylor series for a function that is a polynomial?

Section 8.9

1. What is "Euler's Identity"?
2. What is the purpose of Figure 8.15?
3. Why would Section 8.8 be meaningless if we did not discuss Section 8.9?

Section 8.10

1. Find the first four terms of the binomial series for $f(x) = (1 + x)^{1/3}$.
2. Find the first eight terms of the binomial series for $f(x) = (1 + x)^3$.
3. Compare and contrast your answers to the first two questions.

Section 9.1

1. What are the Cartesian coordinates for the polar point $(3, \frac{3\pi}{2})$?
2. What does the polar curve $r = 10$ look like?
3. What does the polar curve $\theta = \frac{\pi}{3}$ look like?

Section 9.2

1. What shape is a "cardiod" curve?
2. What shape is a "lemniscate" curve?
3. What is the symmetry test to determine if a polar curve has symmetry about the origin?

Section 9.3

1. How does the determination of the slope of a tangent line differ when the curve is described using polar coordinates?
2. How does the determination of the area enclosed by a curve differ when the curve is described using polar coordinates?
3. How does the determination of the arc length of a curve differ when the curve is described using polar coordinates?