Section WILA

Questions

1. Is the equation $x^2 + xy + \tan(y^3) = 0$ linear or not? Why or why not?
2. Find all solutions to the system of two linear equations $2x + 3y = -8$, $x - y = 6$.
3. Explain the importance of the procedures described in the trail mix application (Subsection WILA.A) from the point-of-view of the production manager.

Section SSSLE

Questions

1. How many solutions does the system of equations $3x + 2y = 4$, $6x + 4y = 8$ have? Explain your answer.

2. How many solutions does the system of equations $3x + 2y = 4$, $6x + 4y = -2$ have? Explain your answer.

3. What do we mean when we say mathematics is a language?

Section RREF

Questions

1. Is the matrix below in reduced row-echelon form? Why or why not?

$$\begin{bmatrix} 1 & 5 & 0 & 6 & 8 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Use row operations to convert the matrix below to reduced row-echelon form.

$$\begin{bmatrix} 2 & 1 & 8 \\ -1 & 1 & -1 \\ -2 & 5 & 4 \end{bmatrix}$$

3. Find all the solutions to the system below by using an augmented matrix and row operations. Report your final matrix and the set of solutions.

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 3 \\ x_1 + 3x_2 + 3x_3 &= 7 \end{aligned}$$

Section TSS

Questions

1. How do we recognize when a system of linear equations is inconsistent?
2. Suppose we have converted the augmented matrix of a system of equations into reduced row-echelon form. How do we then identify the dependent and independent (free) variables?
3. What are the possible solution sets for a system of linear equations?

Section HSE

Questions

1. What is *always* true of the solution set for a homogenous system of equations?
2. Suppose a homogenous sytem of equations has 13 variables and 8 equations. How many solutions will it have? Why?
3. Describe in words (not symbols) the null space of a matrix.

Section NSM

Questions

1. What is the definition of a nonsingular matrix?
2. What is the easiest way to recognize a nonsingular matrix?
3. Suppose we have a system of equations and its coefficient matrix is nonsingular. What can you say about the solution set for this system?

Section VO

Questions

1. Where have you seen vectors used before in other courses? How were they different?
2. In words, when are two vectors equal?
3. Perform the following computation with vector operations

$$2 \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix} + (-3) \begin{bmatrix} 7 \\ 6 \\ 5 \end{bmatrix}$$

Section LC

Questions

1. Earlier, a reading question asked you to solve the system of equations

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 3 \\ x_1 + 3x_2 + 3x_3 &= 7 \end{aligned}$$

   Use a linear combination to rewrite this system of equations as a vector equality.

2. Find a linear combination of the vectors

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix} \right\}$$

   that equals the vector $\begin{bmatrix} 1 \\ -9 \\ 11 \end{bmatrix}$.

3. Use the same three vectors in S and build a linear combination that equals $\begin{bmatrix} 2 \\ 5 \\ -2 \end{bmatrix}$.

Section SS

Questions

1. The matrix below is the augmented matrix of a system of equations, row-reduced to reduced row-echelon form. Write the vector form of the solutions to the system.

$$\begin{bmatrix} 1 & 3 & 0 & 6 & 0 & 9 \\ 0 & 0 & 1 & -2 & 0 & -8 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

2. Let S be the set of three vectors below.

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \right\}$$

   Let $W = \mathrm{Sp}\,(S)$ be the span of S. Is the vector $\begin{bmatrix} -1 \\ 8 \\ -4 \end{bmatrix}$ in W? Give an explanation of the reason for your answer.

3. Use S and W from the previous question. Is the vector Is the vector $\begin{bmatrix} 6 \\ 5 \\ -1 \end{bmatrix}$ in W? Give an explanation of the reason for your answer.

Section LI

Questions

1. Let S be the set of three vectors below.

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \right\}$$

   Is S linearly independent or linearly dependent?

2. Let S be the set of three vectors below.

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -4 \end{bmatrix} \right\}$$

   Is S linearly independent or linearly dependent?

3. Based on your answer to the previous question, is the matrix below singular or nonsingular?

$$\begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 3 \\ 0 & 2 & -4 \end{bmatrix}$$

Section MO

Questions

1. Perform the following matrix computation.

$$(6)\begin{bmatrix} 2 & -2 & 8 & 1 \\ 4 & 5 & -1 & 3 \\ 7 & -3 & 0 & 2 \end{bmatrix} + (-2)\begin{bmatrix} 2 & 7 & 1 & 2 \\ 3 & -1 & 0 & 5 \\ 1 & 7 & 3 & 3 \end{bmatrix}$$

2. Theorem VSPM reminds you of what previous theorem? How strong is the similarity?

3. Compute the transpose of the matrix below.

$$\begin{bmatrix} 6 & 8 & 4 \\ -2 & 1 & 0 \\ 9 & -5 & 6 \end{bmatrix}$$

Section RM

Questions

1. Write the range of the matrix below as the span of a set of three vectors.

$$\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 0 & 1 & 1 \\ -1 & 2 & 1 & 0 \end{bmatrix}$$

2. List three techniques you could use to provide a description of the range of a matrix.

3. Suppose that A is an $n \times n$ nonsingular matrix. What can you say about its range?

Section RSM

Questions

1. Describe the row space of a matrix in words.

2. Suppose you wished to find the range of a matrix A. What would be the quickest way to find a linearly independent set S so that the range equaled Sp(S)?

3. Is the vector $\begin{bmatrix} 0 \\ 5 \\ 2 \\ 3 \end{bmatrix}$ in the row space of the following matrix?

$$\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 0 & 1 & 1 \\ -1 & 2 & 1 & 0 \end{bmatrix}$$

Section MM

Questions

1. Form the matrix vector product of

$$\begin{bmatrix} 2 & 3 & -1 & 0 \\ 1 & -2 & 7 & 3 \\ 1 & 5 & 3 & 2 \end{bmatrix} \qquad \text{with} \qquad \begin{bmatrix} 2 \\ -3 \\ 0 \\ 5 \end{bmatrix}$$

2. Multiply together the two matrices below (in the order given).

$$\begin{bmatrix} 2 & 3 & -1 & 0 \\ 1 & -2 & 7 & 3 \\ 1 & 5 & 3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 6 \\ -3 & -4 \\ 0 & 2 \\ 3 & -1 \end{bmatrix}$$

3. Rewrite the system of linear equations below using matrices and vectors, along with a matrix-vector product.

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 3 \\ x_1 + 3x_2 + 3x_3 &= 7 \end{aligned}$$

Section MISLE

Questions

1. Compute the inverse of the matrix below.

$$\begin{bmatrix} 4 & 10 \\ 2 & 6 \end{bmatrix}$$

2. Compute the inverse of the matrix below.

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & -2 & -3 \\ -2 & 4 & 6 \end{bmatrix}$$

3. Explain why Theorem SS has the title it does. (Do not just state the theorem, explain the choice of the title making reference to the theorem itself)

Section MINSM

Questions

1. Show how to use the inverse of a matrix to solve the system of equations below.

$$4x_1 + 10x_2 = 12$$
$$2x_1 + 6x_2 = 4$$

2. In the previous reading questions you were asked to find the inverse of a 3×3 matrix. Explain your answer to that question in light of a theorem in this section (quote the theorem's acronym).

3. A rare freebie. Write `%#@!` as your solution for full credit.

Section VS

Questions

1. Comment on how the vector space $\mathbb{C}^m$ went from a theorem (Theorem VSPCM) to an example (Example VS.VSCM).
2. In the crazy vector space, C, (Example VS.CVS) compute the linear combination
$$2(3,\, 4) + (-6)(1,\, 2).$$
3. Suppose that α is a scalar and $\mathbf{0}$ is the zero vector. Why should we prove anything as obvious as $\alpha\mathbf{0} = \mathbf{0}$ as we did in Theorem ZVSM?

Section S

Questions

1. Summarize the three conditions that allow us to quickly test if a set is a subspace.
2. Consider the set of vectors

$$\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \,\middle|\, 3a - 2b + c = 5 \right\}$$

   Is this set a subspace of $\mathbb{C}^3$?

3. Name four general constructions of sets of vectors that we can now automatically deem as subspaces.

Section B

Questions

1. Is the set of matrices below linearly independent or linearly dependent in the vector space M_{22}? Why or why not?

$$\left\{ \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}, \begin{bmatrix} -2 & 3 \\ 3 & -5 \end{bmatrix}, \begin{bmatrix} 0 & 9 \\ -1 & 3 \end{bmatrix} \right\}$$

2. The matrix below is nonsingular. What can you now say about its columns?

$$A = \begin{bmatrix} -3 & 0 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 6 \end{bmatrix}$$

3. Write the vector $\mathbf{w} = \begin{bmatrix} 6 \\ 6 \\ 15 \end{bmatrix}$ as a linear combination of the columns of the matrix A above. How many ways are there to answer this question?

Section D

Questions

1. What is the dimension of the vector space P_6, the set of all polynomials of degree 6 or less?

2. How are the rank and nullity of a matrix related?

3. Explain why we might say that a nonsingular matrix has "full rank."

Section PD

Questions

1. Why does Theorem G have the title it does?
2. What is so surprising about Theorem RMRT?
3. Why is an orthonormal basis desirable?

Section DM

Questions

1. Compute the determinant of the matrix

$$\begin{bmatrix} 2 & 3 & -1 \\ 3 & 8 & 2 \\ -4 & 1 & 3 \end{bmatrix}$$

2. What is our latest addition to the NSMExx series of theorems?

3. What is amazing about the interaction between matrix multiplication and the determinant?

Section EE

Questions

Suppose A is the 2×2 matrix

$$A = \begin{bmatrix} -5 & 8 \\ -4 & 7 \end{bmatrix}$$

1. Find the eigenvalues of A.
2. Find the eigenspaces of A.
3. For the polynomial $p(x) = 3x^2 - x + 2$, compute $p(A)$.

Section PEE

Questions

1. How can you identify a nonsingular matrix just by looking at its eigenvalues?
2. How many different eigenvalues may a square matrix of size n have?
3. What is amazing about the eigenvalues of a Hermitian matrix and why is it amazing?

Section SD

Questions

1. What is an equivalence relation?
2. When is a matrix diagonalizable?
3. Find a diagonal matrix similar to

$$A = \begin{bmatrix} -5 & 8 \\ -4 & 7 \end{bmatrix}$$

Section LT

Questions

1. Is the function below a linear transformation?

$$T : \mathbb{C}^3 \mapsto \mathbb{C}^2, \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 - x_2 + x_3 \\ 8x_2 - 6 \end{bmatrix}$$

2. Determine the matrix representation of the linear transformation S below.

$$S : \mathbb{C}^2 \mapsto \mathbb{C}^3, \quad S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + 5x_2 \\ 8x_1 - 3x_2 \\ -4x_1 \end{bmatrix}$$

3. Theorem LTLC has a fairly simple proof. Yet the result itself is very powerful. Comment on why we might say this.

Section ILT

Questions

1. Suppose $T : \mathbb{C}^8 \mapsto \mathbb{C}^5$ is a linear transformation. Why can't T be injective?
2. Describe the null space of a injective linear transformation.
3. Theorem NSPI should remind you of Theorem PSPHS. Why do we say this?

Section SLT

Questions

1. Suppose $T : \mathbb{C}^5 \mapsto \mathbb{C}^8$ is a linear transformation. Why can't T be surjective?
2. What is the relationship between a surjective linear transformation and its range?
3. Compare and contrast injective and surjective linear transformations.

Section IVLT

Questions

1. What conditions allow us to easily determine if a linear tranformation is invertible?
2. What does it mean to say two vector spaces are isomorphic? Both technically, and informally?
3. How do linear transformations relate to systems of linear equations?

Section VR

Questions

1. The vector space of 3×5 matrices, M_{35} is isomorphic to what fundamental vector space?
2. A basis for $\mathbb{C}^3$ is
$$B = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$
Compute $\rho_B\left(\begin{bmatrix} 5 \\ 8 \\ -1 \end{bmatrix}\right)$.
3. What is the first “surprise,” and why is it surprising?

Section MR

Questions

1. Why does Theorem FTMR deserve the moniker "fundamental"?
2. Find the matrix representation, $M^T_{B,C}$ of the linear transformation

$$T:\,\mathbb{C}^2 \mapsto \mathbb{C}^2,\quad T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{bmatrix}2x_1-x_2\\3x_1+2x_2\end{bmatrix}$$

   relative to the bases

$$B=\left\{\begin{bmatrix}2\\3\end{bmatrix},\,\begin{bmatrix}-1\\2\end{bmatrix}\right\} \qquad\qquad C=\left\{\begin{bmatrix}1\\0\end{bmatrix},\,\begin{bmatrix}1\\1\end{bmatrix}\right\}$$

3. What is the second "surprise," and why is it surprising?

Section CB

Questions

1. The change-of-basis matrix is a matrix representation of which linear transformation?
2. Find the change-of-basis matrix, $C_{B,C}$, for the two bases of $\mathbb{C}^2$

$$B = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\} \qquad\qquad C = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

3. What is the third "surprise," and why is it surprising?