Proofs that Really Count: The Art of Combinatorial Proof. *By Arthur T. Benjamin and Jennifer J. Quinn.* Mathematical Association of America, Washington, DC, 2003. ISBN 0-88385-333-7.

To a combinatorialist some of the most pleasing proofs use the following standard technique. Construct a finite set and count its elements in two very different ways. The two expressions obtained must then be equal and so an identity can be formulated, often linking two very different looking expressions. To the authors, both accomplished combinatorialists, this is the "*only* right [type]" of proof. So this is the theme of the book — the 246 identities described are typically proved by this technique, and if not, counting is always employed. It is proficiency at exploiting this technique that gives rise to the use of the word "Art" in the subtitle. (The publisher has rumored that they will subtitle the sequel as "The Jenny of Combinatorial Proof.")

Perhaps the first example that comes to mind of using this technique is the identity $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. To prove this, construct all of the subsets of a set with n elements. Count the possibilities by deciding, element by element, if each element is in the set or not, to get 2^n possible subsets. Now, partition the set of all subsets according to their size (a technique deemed "conditioning" in the book). Each part then contains $\binom{n}{k}$ subsets, and their total gives the sum in the second expression. One would think this might be the prototypical example as a lead-off for Chapter 1. Instead, the book begins with Fibonacci numbers and the many identities that relate them to binomial coefficients and each other. Many of these proofs use linear strips of squares and dominoes as the basis for the sets to be counted. Binomial coefficients do not take on a starring role until Chapter 5, where the identity described above occurs as Identity 128 (is the power of 2 here just a coincidence?).

Each chapter contains about 10 identities, each proved carefully, along with a few more general theorems that are necessary along the way. The chapters are meant to be independent of each other, so the book can be read in a nonlinear fashion. Each chapter has roughly 10 identities left to the reader as exercises, with detailed hints and solutions for all in an appendix. Additionally, several chapters have a few "Uncounted Identities," which are unnumbered and not included in the count of the 246 identities mentioned above. These are identities that relate combinatorial quantities in a natural way, yet are still in need of the "right" type of proof. The nine chapters are titled: Fibonacci Identities; Gibonacci and Lucas Identities; Linear Recurrences; Continued Fractions; Binomial Identities; Alternating Sign Binomial Identities; Harmonic and Stirling Number Identities; Number Theory; Advanced Fibonacci & Lucas Identities.

This book is not meant as a textbook, though a capable undergraduate, and most any graduate student, could learn much about combinatorics by working through it. Indeed, there appears to be significant contributions by undergraduates to its creation. It will perhaps be most useful as a resource for those teaching combinatorics, discrete mathematics, or even number theory, from a different text. While not meant to be comprehensive or encyclopedic, the appendix listing all 246 identities makes it a useful reference for that not-too-obscure identity that you cannot quite recall exactly.

This book is written in an engaging, conversational style and this reviewer found it enjoyable to read through (besides learning a few new things). Along the way there are a few surprises, like the "world's fastest proof by induction" and a magic trick. As a resource for teaching, and a handy basic reference, it will be a great addition to the library of anyone that uses combinatorial identities in their work.

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