

The Art of Mathematics: Coffee Time in Memphis By Béla Bollobás. Cambridge University Press, New York, 2006. \$34.99. xv+359 pp., softcover. ISBN 0-521-693950.

A common technique of reviewers writing for *Mathematical Reviews* is to simply quote the abstract of the research article being reviewed. A similar shortcut, where this reviewer might simply quote the Preface of this book, could suffice as concise and accurate review of the book. Instead, the curious may sign on to Amazon.com and employ the “Search Inside This Book” feature with “preface” as the search term, and read the three pages of the full preface there.

The Preface describes this book as a collection of problems the author thinks Paul Erdős or J.E. Littlewood would have found enticing. As the title suggests, its genesis is an informal problem-solving group that met at the University of Memphis. There are 157 problems in the front section (the first 28 can be seen at Amazon.com in the “Excerpt”), mostly stated in an informal fashion. A short section of hints follows, and then the final 315 pages contain full solutions written as proofs, often to reformulations of the problems in more formal terms. The serious nature of the mathematics is counterbalanced by lighthearted artwork by the author’s wife.

It would be pointless to try to entirely classify the nature of the problems. Perhaps a majority have roots in discrete mathematics, but many are related to geometry, topology or analysis. The author hopes that the problems could be used to “inspire undergraduates,” and there are problems that are appropriate for strong, but not exceptional, undergraduates. However, many of the problems require a lot of machinery that even a strong undergraduate will not have seen before. There is no indicated gradation to the problems in difficulty, or organization by the mathematics brought to bear. The author makes no apology for this, saying the collection is “haphazard” and is meant to be sampled rather than read from front-cover to back-cover. The first and penultimate problems might illustrate the range.

Problem Number 1 is due to Rado and popularized by Littlewood.

A lion and a Christian in a closed circular arena have equal maximum speeds. What tactics should the lion employ to be sure of his meal? In other words, can the lion catch the Christian in finite time?

The solution contains a journal reference and discusses generalizations such as n lions in an n -dimensional ball, and two lions in a bounded area with rectifiable lakes.

While the first problem is easy to state and understand, problem Number 156 has a more technical statement.

Let $x_1, x_2, x_3, \dots, x_n$ be vectors of norm at most 1 in a d -dimensional normed space such that $x_1 + x_2 + x_3 + \dots + x_n = 0$. Show that there is a permutation $\pi \in S_n$ such that

$$\|x_{\pi(1)} + x_{\pi(2)} + x_{\pi(3)} + \dots + x_{\pi(k)}\| \leq d$$

for every k .

The hint includes a reformulation as a more general problem, and the proof in the solutions is followed by seventeen references, with several from the 1980’s and 90’s and one from 2000.

While both of these problems have proofs that might escape an undergraduate student, there are some problems appropriate for students, such as Number 40, which requires an edifying application of induction.

Cut out a square of a 2^n by 2^n chess board. Show that the remaining $2^{2n} - 1$ squares can be tiled with L -tiles where an L -tile is a union of three squares sharing a vertex.

Besides the memory of Erdős and Littlewood, the author’s main criteria for selecting problems is that they should be “mathematics with fun.” In this, the author has succeeded, and every mathematician should find something fun, novel and unexpected, alongside some old friends. This book is a must-have for the problem solver, and would be a valuable addition to any personal library. Those who lead problem-solving seminars (such as preparation for the Putnam Mathematical Competition) will find additional material here. Finally, for an academic library this book provides a bridge between purely recreational mathematics and pure mathematics.

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